Math 10 Lecture Videos

Section 3.3: Slope

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OBJECTIVES:

1. Compute a line’s slope.
2. Use slope to show that lines are parallel.
3. Use slope to show that lines are perpendicular.
4. Calculate the rate of change in applied situations.

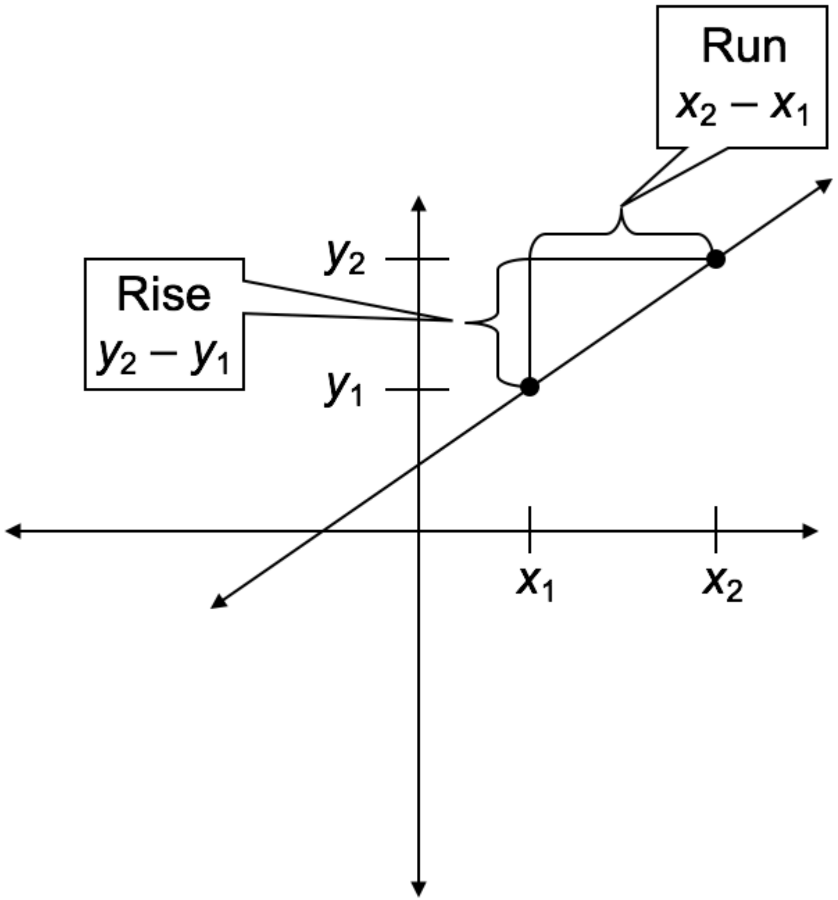
Objective 1: Compute a Line's Slope

Slope Defined:

- A measure of the steepness of a line.
- The slope of the line through the distinct points (x1,y1) and (x2, y2) is:

$$m = \frac{Change\ in\ y}{Change\ in\ x} = \frac{Rise}{Run} = \frac{y_2 - y_1}{x_2 - x_1}$$

where $x_2 - x_1 \neq 0$



Objective 1: Compute a Line's Slope

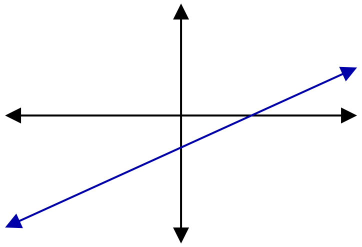
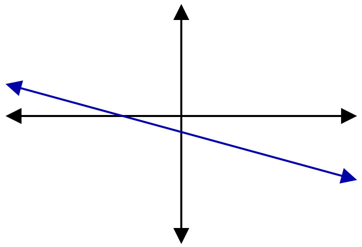
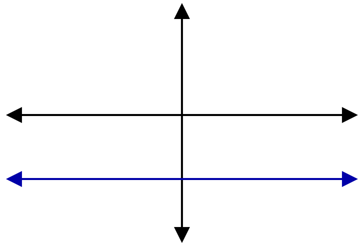
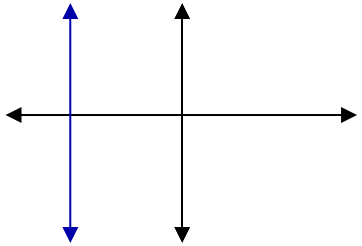
Example 1: Find the slope of the line passing through the pair of points (-7,-8) and (4,-2). Then explain what the slope means.

Let $(x_1,y_1) = (-7,-8)$ and let $(x_2,y_2) = (4,-2)$

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-8)}{4-(-7)} = \frac{-2+8}{4+7} = \frac{6}{11}$$

This means that for every 6 units that the line (that passes through both points) travels up, it also travels 11 units to the right.

Objective 1: Compute a Line's Slope

POSSIBILITIES FOR A LINE'S SLOPE	
Positive Slope	**Negative Slope**
	
Zero Slope	**Undefined Slope**
	

Objective 1: Compute a Line’s Slope

Example 1: Find the slope of the line passing through the pair of points (4,-2) and (-1,5). Then explain what the slope means.

Let $(x_1,y_1) = (4,-2)$ and let $(x_2,y_2) = (-1,5)$

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-2)}{-1-4} = \frac{7}{-5} = -\frac{7}{5}$$

Since the slope is negative, the line falls from left to right.

Objective 1: **Compute a Line's Slope**

Example 2: Find the slope of the line passing through the pair of points (6,5) and (2,5). Then explain what the slope means.

Let $(x_1,y_1) = (6,5)$ and let $(x_2,y_2) = (2,5)$

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_1 - y_1}{x_2 - x_1} = \frac{5-5}{2-6} = \frac{0}{-4} = 0$$

Since the slope is 0, the line is horizontal.

Objective 1: **Compute a Line's Slope**

Example 3: Find the slope of the line passing through the pair of points (1,6) and (1,4). Then explain what the slope means.

Let $(x_1,y_1) = (1,6)$ and let $(x_2,y_2) = (1,4)$

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-6}{1-1} = \frac{-2}{0}$$

Because division by 0 is undefined, the slope is also undefined, which makes the line vertical.

Objective 2: Use slope to show that lines are parallel.

Slope and Parallel Lines
1. If two non-vertical lines are parallel, then they have the same slope.
2. If two distinct non-vertical lines have the same slope, then they are parallel.
3. Two distinct vertical lines, each with undefined slope, are parallel.

Objective 2: Use slope to show that lines are parallel.

Two lines are parallel if they have the same slope.

The following lines are parallel:

$y = 2x + 6$ and $y = 2x - 4$ are parallel.

$y = -4x + 5$ and $y = -4x + 3$ are parallel.

Objective 2: Use slope to show that lines are parallel.

Example: Show that the line passing through (4,2) and (6,6) is parallel to the line passing through (0,-2) and (1,0)

Slope of line through (4,2) and (6,6):

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{6-2}{6-4} = \frac{4}{2} = 2$$

Slope of line through (0,-2) and (1,0):

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{0-(-2)}{1-0} = \frac{2}{1} = 2$$

Since their slopes are equal, the lines are parallel.

Objective 3: Use slope to show that lines are perpendicular.

Slope and Perpendicular Lines
1. If two non-vertical lines are perpendicular, then the product of their slopes is -1.
2. If the product of the slopes of two lines is -1, then the lines are perpendicular.
3. A horizontal line having zero slope is perpendicular to a vertical line having undefined slope.

Objective 3: Use slope to show that lines are perpendicular.

One line is perpendicular to another line if its slope is the negative reciprocal of the slope of the other line.

The following lines are perpendicular:

$y = 2x + 6$ and $y = -\frac{1}{2}x - 4$ are perpendicular.

$y = -4x + 5$ and $y = \frac{1}{4}x + 3$ are perpendicular.

Objective 3: Use slope to show that lines are perpendicular.

Example: Show that the line passing through (-1,4) and (3,2) is perpendicular to the line passing through (-2,-1) and (2,7)

Line passing through (-1,4) and (3,2):

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{2-4}{3-(-1)} = \frac{-2}{4} = -\frac{1}{2}$$

Line passing through (-2,-1) and (2,7):

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{7-(-1)}{2-(-2)} = \frac{8}{4} = 2$$

Since the product of their slopes is $-\frac{1}{2}(2) = -1$, the lines are perpendicular

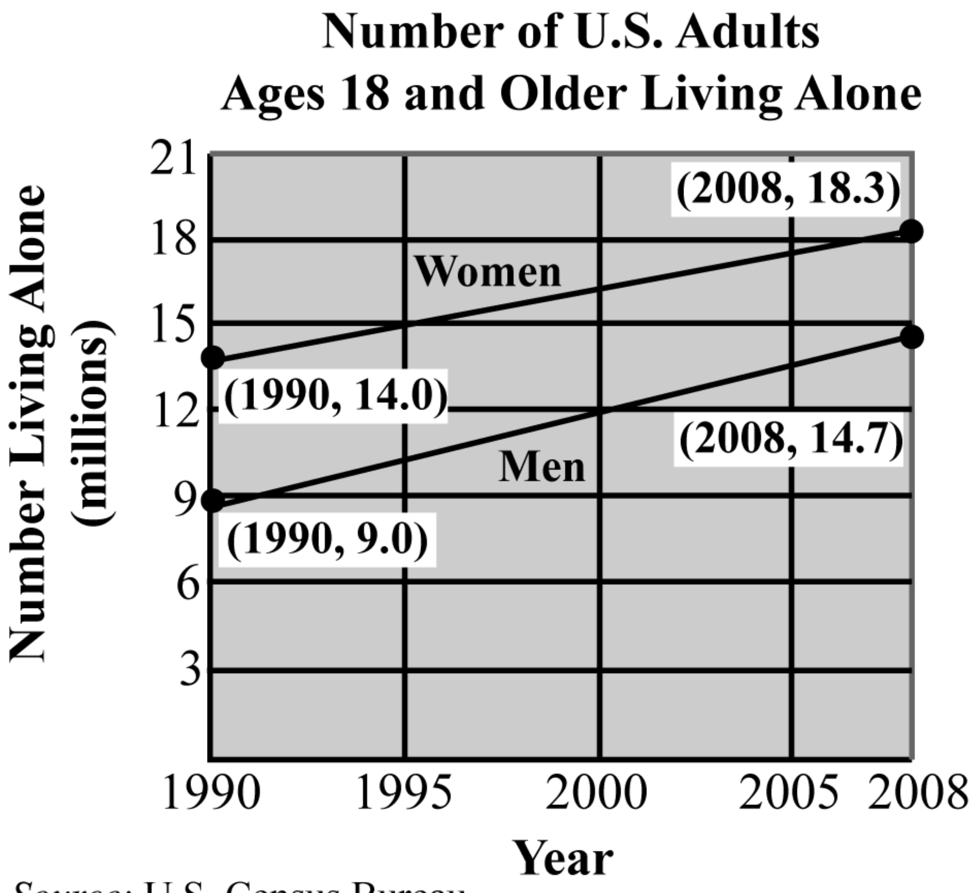
Objective 4: Calculate the rate of change in applied situations.

- Slope is defined as the ratio of a change in *y* to a corresponding change in *x*.
- It tells how fast *y* is changing with respect to *x*.
- The slope of a line represents its **rate of change**.

Objective 4: Calculate the rate of change in applied situations.

Example:

Use the ordered pairs in the figure shown to find the slope of the line segment that represents men. Express the slope correct to two decimal places and describe what it represents.



Source: U.S. Census Bureau

Objective 4: Calculate the rate of change in applied situations.

Let $(x_1, y_1) = (1990, 9.0)$ and $(x_2, y_2) = (2008, 14.7)$.

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14.7 - 9.0}{2008 - 1990} = \frac{5.7}{18} \approx 0.32$$

The number of men living alone increased at a rate of 0.32 million per year.

The rate of change is 0.32 million men per year.

OBJECTIVES:

1. Compute a line’s slope. ✓
2. Use slope to show that lines are parallel. ✓
3. Use slope to show that lines are perpendicular. ✓
4. Calculate the rate of change in applied situations. ✓